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(6) THE PERFORMANCE CHARACTERISTICS OF TWO  
EXTENSIONS OF THE SIGN DETECTOR.

by

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Abstract

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Two related techniques have been proposed in the past for improving the performance of the sign detector, through higher-order data quantization. The fixed-threshold m-interval detector and the generalized sign detector using a conditional test are both nonparametric detectors which are fairly simple to implement. In this paper we compare the asymptotic and finite-sample, finite-signal performance characteristics of these two detectors, and point out their relative advantages and disadvantages.

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## I. INTRODUCTION

For the detection of a deterministic signal in additive noise with zero median, the sign detector is an easily implemented nonparametric detector with a constant probability of type I error (false alarm). Let  $\underline{X} = (X_1, X_2, \dots, X_n)$  be the vector of possibly signal-bearing observations, so that

$$X_i = W_i + \theta s_i, \quad i=1, 2, \dots, n, \quad (1)$$

where  $\theta \geq 0$  and the  $W_i$  are independent random variables representing noise. We will denote by  $F$  and  $f$  the common probability distribution and density functions, respectively, of the  $W_i$ . The signal components  $s_i$  are assumed to be non-zero.

In the constant signal case the sign detector test statistic for testing  $\theta = 0$  vs.  $\theta > 0$  is

$$T = \sum_{i=1}^n \text{sgn}(X_i) \quad (2)$$

where

$$\text{sgn}(x) = \begin{cases} 1, & x > 0 \\ -1, & x \leq 0 \end{cases}$$

For a time-varying signal, the above statistic can be modified to

$$\tilde{T} = \sum_{i=1}^n \text{sgn}(s_i) \text{sgn}(X_i) \quad (3)$$

or to

$$\hat{T} = \sum_{i=1}^n s_i \text{sgn}(X_i) \quad (4)$$

The statistic  $\tilde{T}$  is simpler to implement than  $\hat{T}$ , but has a somewhat lower efficiency, as will be seen later.

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The performance of the sign detector is rather poor for noise densities such as the Gaussian, when compared to the corresponding optimal (parametric) detectors and to more complex nonparametric detectors based on data ranking. In order to obtain improved performance over that of the sign detector, without a major increase in complexity, two schemes have been proposed based on a higher-order partitioning of the observations. The m-interval detector has been described in [1], and the generalized sign detector was first formulated in [2]. In this paper we will compare the relative performances and characteristics of these two detectors. We will consider the asymptotic performance of the detectors (for  $\theta \rightarrow 0$  and  $n \rightarrow \infty$ ), as well as finite-sample characteristics. Both of these extensions of the simple sign detector have also been applied in schemes for the detection of random signals, in multi-input systems, for sequential detection, and in other applications ([3] - [6]). However, the comparisons we will present for the deterministic signal case with a single fixed-length observation vector will bring out the main features of these two schemes.

In the next section we describe briefly the m-interval and generalized sign detectors, before considering their performance characteristics in Section III.

## II. EXTENSIONS OF THE SIGN DETECTOR

Let us first assume that the signal in (1) is constant, so that  $s_i = s > 0$ ,  $i=1,2,\dots,n$ . Consider a vector  $\underline{a} = (a_0, a_1, \dots, a_m)$ , where  $a_0 = -\infty$ ,  $a_m = \infty$  and  $a_j > a_{j-1}$ ,  $j=1,2,\dots,m$ . We define intervals

$$I_j = (a_{j-1}, a_j], \quad j=1,2, \dots, m \quad (5)$$

and denote by  $N_j$  the number of observations  $X_i$  falling in  $I_j$ . Now suppose that the  $a_j$  are quantiles of the noise distribution  $F$ , so that  $F(a_j) = j/m$ . In this case the test statistic defined by

$$Q = \sum_{j=1}^m b_j N_j, \quad (6)$$

where  $\underline{b} = (b_1, b_2, \dots, b_m)$  is a deterministic weight vector, can be used to form a nonparametric test for  $\theta=0$  vs.  $\theta>0$  in (1). This is because for any noise distribution function  $F$  with quantiles given by  $\underline{a}$ , the distribution of  $Q$  when  $\theta=0$  is exactly known. A detector formed according to this scheme is known as an  $m$ -interval detector [1]. Note that the sign detector is obtained when  $m=2$ ,  $a_1=0$  and  $b_1=-1$ ,  $b_2=1$ . It is also clear that  $\underline{a}$  may be any vector of partitioning parameters  $a_j$  for which the  $F(a_j) = p_j$  are known. It may be more reasonable to assume that the  $\underline{a}$  vector corresponding to the values  $p_j=j/m$  can be estimated from prior data. The weight vector  $\underline{b}$  would normally be picked to result in a good compromise in performance over some collection of possible density functions  $f$ .

Let us now assume that the noise density function is symmetric, so that  $f(x) = f(-x)$ . Let the vector  $\underline{a}$  have odd-symmetry, that is,  $a_{m-\ell} = -a_\ell$ ,  $\ell=1, 2, \dots, m-1$ . If  $f$  is symmetric, an odd-symmetric  $\underline{a}$  would be required for the  $m$ -interval detector if  $p_j = j/m$ . In addition, as will be seen in the next section, the optimum choice of  $\underline{a}$  for given symmetric  $f$  is odd-symmetric.

The generalized sign detector [2] based on any odd-symmetric  $\underline{a}$  vector achieves a fixed type I error probability for any symmetric density function of the noise. Its test statistic is also defined by (6), but the test is formed by a comparison with a variable threshold, with a variable randomization probability when  $Q$  is equal to the threshold. Note that knowledge only of the symmetry of  $f$  is not sufficient to yield the distribution of  $Q$  when  $\theta = 0$ . Consider, however, the statistic  $\underline{C} = (C_1, C_2, \dots, C_p)$  with

$$C_\ell = N_\ell + N_{m-\ell} + 1, \quad \ell = 1, 2, \dots, p, \quad (7)$$

where  $p = [m/2]$ , the largest integer less than  $m/2$ . Now conditioned on  $\underline{C}$ , the distribution of  $Q$  when  $\theta = 0$  is completely defined; the distribution of  $N_\ell$  given  $C_\ell$  is binomial, independent of the binomial distribution of  $N_j$  given  $C_j$  for  $j \neq \ell$ . Thus a nonparametric test can be implemented with

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thresholds and randomization probabilities computed as functions of  $C$ . As an example, for a four-level test ( $m=4$ ) the test parameters are functions of the scalar  $C_1 = N_1 + N_4$ . It should be noted that here also the case  $m=2$  reduces to the sign test. This detector structure is shown in Fig. 1.

The  $m$ -interval detector is based on the assumption that the vector of quantiles  $\underline{a}$  for the noise is known, or at least that the  $F(a_j)$  values are known for the components of a given  $\underline{a}$ , for the class of allowable noise densities. Such classes of noise densities are nonparametric classes, but are somewhat restrictive in their membership. For example, for  $m=3$  or 4 with an odd-symmetric  $\underline{a}$  vector of quantiles, only one Gaussian density can belong to the allowable class. For  $m=5$  and some assumed quantile vector  $\underline{a}$ , no Gaussian density (or any other parametric density with only one or two free parameters) may belong to the allowable class. The generalized sign detector needs a conditional structure which represents a slight increase in implementation cost, but gives nonparametric performance for the large class of symmetric densities for the noise. The increase in complexity due to the conditional structure is usually nominal; for the case where  $m=3$  or 4, conditioning is only on the scalar  $C_1$ , and can be easily implemented using a table look-up scheme [2]. An increase in  $m$  from two to four generally represents a practical trade-off between improved performance and increase in detector complexity for both the conditional and unconditional implementations of the  $m$ -interval detector.

Since the  $m$ -interval detector is based on more specific assumptions about the class of allowable noise densities, its detection performance can be expected to be better than that of the conditional-test implementation leading to the generalized sign detector. It will be found, however, that the difference in performance is small for most cases of interest. Before we take up the performance comparisons, we will briefly indicate

how both detectors above may be modified for the case where the  $s_i$ ,  $i=1,2,\dots,n$ , are not all equal.

Note that the test statistic  $Q$  of (6) can be written as [7]

$$Q = \sum_{j=1}^m b_j \sum_{i=1}^n Z_{ij}, \quad (8)$$

where

$$Z_{ij} = \begin{cases} 1, & X_i \in I_j \\ 0, & \text{otherwise} \end{cases}, \quad (9)$$

so that  $N_j = \sum_{i=1}^n Z_{ij}$ . Simple modifications of  $Q$  which lead to more efficient tests for non-constant signals may be defined as the test statistics

$$\tilde{Q} = \sum_{j=1}^m b_j \sum_{i=1}^n \text{sgn}(s_i) Z_{ij} \quad (10)$$

and

$$\hat{Q} = \sum_{j=1}^m b_j \sum_{i=1}^n s_i Z_{ij} \quad (11)$$

For the  $m$ -interval detector, it is assumed that for  $\theta=0$  the matrix  $\underline{Z}$  of random variables  $Z_{ij}$  has a known distribution. For the generalized sign detector, based on the symmetry of the noise density functions, we also have a known distribution for  $\underline{Z}$  conditioned on  $\underline{C}$  of (7). Thus both  $\tilde{Q}$  and  $\hat{Q}$  can be used for a conditional-test generalized sign detector. It is apparent that  $\tilde{Q}$  and  $\hat{Q}$  reduce to  $\tilde{T}$  and  $\hat{T}$ , respectively, when  $m=2$  and  $b_2 = -b_1 = 1$ .

### III. PERFORMANCE CHARACTERISTICS

#### (a) Asymptotic Performance

The asymptotic relative efficiency (ARE) of two detectors is the ratio of sample-sizes required by the detectors to maintain the same performance (error probabilities) in the limiting case when signal strength approaches zero, and thus sample sizes tend to infinity. In many cases the

ARE of two detectors is simply the ratio of their efficacies [8], the efficacy  $\epsilon$  of a test based on a statistic  $S$  being defined by

$$\epsilon = \lim_{n \rightarrow \infty} \frac{\left[ \frac{d}{d\theta} E\{S\} \Big|_{\theta=0} \right]^2}{n \text{Var} \{S\} \Big|_{\theta=0}} \quad (12)$$

Thus  $\epsilon$  is like a signal-to-noise ratio measure for weak signals.

An interesting conclusion may be drawn from the above. This is that the ARE of the  $m$ -interval detector relative to the generalized sign detector is unity, when  $a$ ,  $b$ , are the same for both detectors, based on any one of the three test statistics of (8), (10), or (11). Although this follows from the fact that the efficacies are then the same for both detectors, some care is needed in proving that this implies an ARE of unity, since the generalized sign detector uses a conditional test. This type of proof has been outlined in [4].

This result implies that for large sample sizes there will be very little difference in performance between the two detectors, even though one is based on a more specific set of assumptions about the symmetric noise density function. This may be intuitively explained by the fact that, in principle, for large sample sizes good estimates of the noise quantiles may be obtained from observed data (signal-bearing or noise-only), given that the noise density is symmetric.

From the definition of (12), it can be shown directly that  $\tilde{\epsilon}$ , the efficacy of the detectors based on  $\tilde{Q}$  of (10), is given by

$$\tilde{\epsilon} = \frac{\sum_{j=1}^m b_j^2 [f(a_{j-1}) - f(a_j)]^2}{\sum_{j=1}^m b_j^2 [F(a_j) - F(a_{j-1})] - \left\{ \sum_{j=1}^m b_j [F(a_j) - F(a_{j-1})] \right\}^2} \quad (13)$$

where  $\overline{s^2} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n s_i^2$ . For  $\hat{Q}$ , the efficacy  $\hat{\epsilon}$  is given by (13) with  $\overline{s^2}$  replaced by  $|\overline{s}|^2 = \lim_{n \rightarrow \infty} \left[ \frac{1}{n} \sum_{i=1}^n |s_i| \right]^2$ . Note that  $|\overline{s}|^2 \leq \overline{s^2}$ . It is easily shown that for given  $\underline{a}$ , the best choice of  $\underline{b}$  maximizing  $\hat{\epsilon}$  or  $\hat{\epsilon}$  is

$$b_j = K \frac{f(a_j-1) - f(a_j)}{F(a_j) - F(a_j-1)}, \quad j=1,2,\dots,m, \quad (14)$$

where  $K>0$  is any constant. The optimum  $\underline{a}$  vector for some assumed noise density  $f$  can then also be obtained; this  $\underline{a}$  is odd-symmetric for symmetric  $f$ .

The vector of quantiles is not necessarily the best choice for  $\underline{a}$  for given  $f$ . However, the quantiles may be reasonable if the detector is to be implemented as a nonparametric detector for the class of allowable noise densities whose quantiles are assumed known based on prior measurements.

#### (b) Finite-Sample Performance

One of the interesting questions which we now consider is the degree of performance degradation in the generalized sign detector as compared to the  $m$ -interval detector, for finite sample sizes and finite signals. The numerical performance results will be obtained for the case where  $m=4$ . Although results for higher-order partitioning can be obtained in principle with no additional programming difficulties, the resulting computation time requirements become rather large. As remarked earlier, the case where  $m=3$  or 4 usually represents a reasonable compromise between improved performance and increased complexity. In any case, the general characteristics of the relative performances of the two detectors will remain the same for larger  $m$ . We will also focus on the constant signal case, although again the time-varying signal case can be analysed with more computation time and will yield similar characteristics.

In obtaining numerical results for detection power, the parameters a and b for the detectors were chosen to yield a good compromise design (based on the analytical efficacy expressions) for the class of noise density functions containing the Gaussian, double-exponential and Cauchy densities. These last two densities are often used in judging the performance of detectors under conditions of heavy-tailed non-Gaussian noise.

For the symmetric densities and odd-symmetric a vector, we will have  $a_0 = -\infty$ ,  $a_2 = 0$ , and  $a_4 = \infty$ , for the case  $m=4$ . For zero-mean Gaussian noise density with variance  $\sigma^2$ , the quantiles  $a_1$  and  $a_3$  are given by  $a_3 = -a_1 = 0.6745\sigma$ . We will now assume that for the  $m$ -interval detector the known (or estimated) quantiles of the symmetric noise distribution are defined by the components of the above a vector with  $a_3 = -a_1 = 0.6745\sigma_0$  where  $\sigma_0$  is a known positive constant. This definition of the parameter a may be thought of as arising from an assumption that the noise density function is nominally Gaussian with variance  $\sigma_0^2$ .

The double-exponential density function is defined by

$$f_e(x) = \frac{1}{\sigma\sqrt{2}} e^{-|x|\sqrt{2}/\sigma}, \quad (15)$$

where  $\sigma^2$  is the variance. In this case the quantiles  $a_1$  and  $a_3$  are given by  $a_3 = -a_1 = 0.4901\sigma$ . Thus  $f_e(x)$  with  $\sigma = 1.376\sigma_0$  has the same quantiles as the Gaussian density with variance  $\sigma_0^2$  for  $m=4$ .

For the Cauchy density function

$$f_c(x) = \frac{1}{\pi k} \frac{1}{1 + (x/k)^2} \quad (16)$$

we have  $a_3 = -a_1 = k$ ; thus  $k = 0.6745\sigma_0$  gives the same quantiles (for  $m=4$ ) as the Gaussian density with variance  $\sigma_0^2$ .

For these symmetric densities, we find from (14) that the optimum  $\underline{b}$  vector has even symmetry, so that  $b_4 = -b_1$  and  $b_3 = -b_2$ . Proceeding to find these asymptotically optimum values as suggested in (a) above, we find that the optimum value of  $\underline{b}$  is defined by  $b_3 = 0.2554b_4$  for Gaussian noise with variance  $\sigma_0^2$ , and by  $b_4 = b_3$  for both the density  $f_e$  with  $\sigma=1.376\sigma_0$  and the density  $f_c$  with  $k=0.6745\sigma_0$ . In all cases the vector  $\underline{a}$  was the common vector of quantiles,  $m=4$ . Note that this says that the sign detector is optimal for the Cauchy density if the partitioning is based on the quantiles for  $m=4$ . The sign detector is also the maximum-efficacy (locally optimum) detector for double-exponential noise.

As a compromise design for the weight-vector  $\underline{b}$ , we will use  $b_3=0.5b_4$ . Together with the quantile vector  $\underline{a}$ , this leads to an efficacy of  $0.82/\sigma_0^2$  for the Gaussian density with variance  $\sigma_0^2$ , an efficacy of  $0.95/\sigma_0^2$  for the double-exponential density with the same quantiles, and an efficacy of  $0.80/\sigma_0^2$  for the Cauchy density with the same quantiles. Note that with the respective optimum analog schemes, the maximum efficacies in these three cases are [7]  $1/\sigma_0^2$ , and  $1.06/\sigma_0^2$  and  $1.1/\sigma_0^2$ . Using a sign detector on these fixed-quantile densities, the efficacies are  $0.64/\sigma_0^2$ ,  $1.06/\sigma_0^2$  and  $0.89/\sigma_0^2$ , respectively. On the other hand, using the linear sum detector which gives the maximum efficacy for Gaussian noise, we get an efficacy of  $0.53/\sigma_0^2$  and zero for  $f_e$  and  $f_c$ , respectively, with the same quantiles. Thus, the compromise design where  $\underline{a}$  is the quantile-vector and  $\underline{b}$  is defined by  $b_3 = 0.5b_4$  is seen to yield a useful design with good overall asymptotic performance compared to the sign detector and the linear detector.

For the vectors  $\underline{a} = (-\infty, -0.6745\sigma_0, 0, 0.6745\sigma_0, \infty)$  and  $\underline{b} = (-2, -1, 1, 2)$ , numerical results on detection powers were obtained for sample sizes  $n=25$  and 50, false-alarm probabilities  $\alpha=10^{-2}$  and  $10^{-3}$  and for a range of values of  $\theta/\sigma_0$  [for constant-signal detection so that  $s_i=1, i=1, \dots, n$ , in (1)] for both the four-interval detector and the four-level generalized sign detector. These results are given in Table I. In this table, detection probabilities are given for the three noise density functions we have discussed previously, each density function having the common quantiles defined above. For each density function detection probabilities are given for the fixed-threshold  $m$ -interval test and the conditional generalized sign test.

Several interesting features are apparent from Table I. As expected, we observe that the conditional test has a power which is smaller than that of the fixed-threshold test. It can be seen that for small values of  $\theta/\sigma_0$ , a condition under which the efficacy of a detector is a good indication of its detection power (for  $n$  not too small), the conditional and fixed-threshold test powers are almost the same. We also find that in this case the variation in detection power between the three different noise density functions is in agreement with the variation in efficacy for these cases. As  $\theta/\sigma_0$  increases, there is a more apparent difference between the powers of the fixed-threshold and conditional tests. However, for the Gaussian case this difference in powers is quite nominal. Compared to the performance of the sign detector, both the conditional and fixed-threshold tests achieve the same high degree of improvement for Gaussian noise.

The double-exponential and Cauchy noise densities seem to lead to somewhat larger differences in powers for the two tests. This may be due to the possibility that knowledge of the quantiles ( $m$ -interval fixed-threshold test implementation) amounts to having more "information", in

the detection context, for the broad-tailed non-Gaussian densities as compared to the Gaussian case, so that larger sample sizes are needed before the conditional and fixed-threshold tests have similar performances.

The detection powers in Table I also reveal that for small signal strength the sign detector performs better for the double-exponential density than the four-level detectors. However, it is interesting that this performance advantage disappears for larger values of  $\theta/\sigma_0$ . This is due to the fact that although the sign detector is locally optimum for double-exponential noise, for non-vanishing  $\theta$  the optimum detector nonlinearity is not the hard limiter, but the "soft" limiter function  $f_e(x-\theta)/f_e(x)$ . Thus for larger values of  $\theta$  the four-level quantizer characteristics of the four-interval fixed-threshold and conditional-threshold detectors give a closer match to the "soft" limiter [7]. For Gaussian noise the four-level detectors are uniformly better than the sign detector, because the uniformly most powerful linear-sum detector is always better approximated by the four-level quantizer characteristic. For the Cauchy density, the variations in relative performance of the four-level and sign detectors are again explained by similar considerations.

Although the fixed-threshold four-interval detector achieves a larger detection power than the conditional threshold detector, it achieves a fixed false-alarm probability provided the noise quantile vector is correctly specified. If the actual noise density function has a quantile vector other than the assumed vector, the false alarm probability  $\alpha$  of the fixed threshold four-level detector will differ from the design value  $\alpha_0$ . The conditional-threshold implementation, on the other hand, retains the design value of  $\alpha$  for any symmetric noise density function.

Table II shows the effect on false alarm probabilities of a mis-match between the design quantile vector  $\underline{a}$  and the actual values of the quantile vector for the noise density function in the fixed-threshold detector. The first four columns of the table show the actual values of  $p_j = F(a_j)$ , where  $F$  is the noise distribution function. The results are shown for two different design values  $\alpha_0$  of the false alarm probability, achieved when  $p_j = 0.25$ ,  $j = 1, 2, 3, 4$ . It is seen that the actual value of  $\alpha$  may differ considerably from the design value, especially for small design values of  $\alpha$ . These results also show that in order to ensure a value for  $\alpha$  which tends to be less than or equal to a value  $\alpha_0$ , the implemented value of the parameter  $a_3 = -a_1$  in  $\underline{a}$  should be taken to be somewhat larger than the nominal or estimated value. This would result in a lower detector power when the nominal assumptions are correct. Table III illustrates this effect for the case of Gaussian noise with variance  $\sigma_0^2$ , where the four-level detection is based on a value  $a_3 = 0.82\sigma_0$  rather than the nominal value  $a_3 = 0.6745\sigma_0$ . In this case it is seen that the conditional test performs almost as well as, or better than, the fixed threshold detector. In fact, the power of the conditional test in Table III is better than in the corresponding situation in Table I. This is because the optimum value for  $a_3$  here is  $0.98\sigma_0$  [7].

The fundamental difference between the  $m$ -interval (fixed-threshold) and generalized sign (conditional test) detectors is that in the latter, the threshold and randomization probability depend on the received data. The conditional test is, nevertheless, fairly easy to implement; one method of implementation is to use a stored set of thresholds and randomizations in a table look-up device, as described in [2]. One simplification which may be desirable in any threshold test is the quantization of the randomization probability, to perhaps only two values zero and 0.5. This would

also reduce storage requirements in the table for a conditional test, only one bit now being required for each randomization probability.

In nonparametric applications the design value of the false alarm probability  $\alpha_0$  would not be exceeded if in each case the exact randomization probability  $r$  is replaced by 0.5 if  $r \geq 0.5$ , and by zero otherwise. In Table IV the effect on  $\alpha$  is shown for such a quantization of  $r$  for both the fixed-threshold and conditional test. These results were obtained with the assumption that the  $\underline{a}$  vector is the vector of quantiles, so that each  $p_j = 0.25$  ( $j=1,2,3,4$ ). The results in Table IV show that the fixed-threshold test achieves a value for  $\alpha$  which is generally, but not always, closer to the design value  $\alpha_0$ .

#### IV. CONCLUSION

Both the fixed-threshold  $m$ -interval detector and the conditional generalized sign detector can yield a significantly better overall detector performance than the simple sign detector. In the limiting case of large samples and vanishing signals both improved detectors have identical performances. The  $m$ -interval detector achieves a slightly larger detection power, but is nonparametric over a restricted class of noise density functions. The generalized sign detector has a fixed alarm probability for any symmetric noise density function, at the expense of a minor increase in implementation complexity. If the quantile vector in the  $m$ -interval detector is imprecisely known, nonparametric performance is possible only with a loss in power. For the generalized sign detector, knowledge of the quantiles is not essential, but serves as a guide in picking a good partitioning scheme for the data.

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FIGURE CAPTION

FIGURE 1    Structure Of Generalized Sign (Conditional) Detector

TABLE CAPTIONS

TABLE I	DETECTION POWER OF $m$ -INTERVAL (FIXED THRESHOLD) AND GENERALIZED SIGN (CONDITIONAL) DETECTORS , $m=4$
TABLE II	ACTUAL TYPE I ERROR PROBABILITIES $\alpha$ OF $m$ -INTERVAL DETECTOR, $m=4$ , FOR DESIGN VALUES $\alpha_0$ , WITH DIFFERENT $p_j$ VALUES FOR THE $\underline{a}$ VECTOR.
TABLE III	EFFECT ON DETECTION POWER IN $m$ -INTERVAL DETECTOR, $m=4$ , OF LARGER $a_3$ PARAMETER TO MAINTAIN $\alpha$ WITHIN DESIGN VALUE $\alpha_0$
TABLE IV	EFFECT ON $\alpha$ OF TWO-LEVEL RANDOMIZATION PROBABILITY (0 AND 0.5)

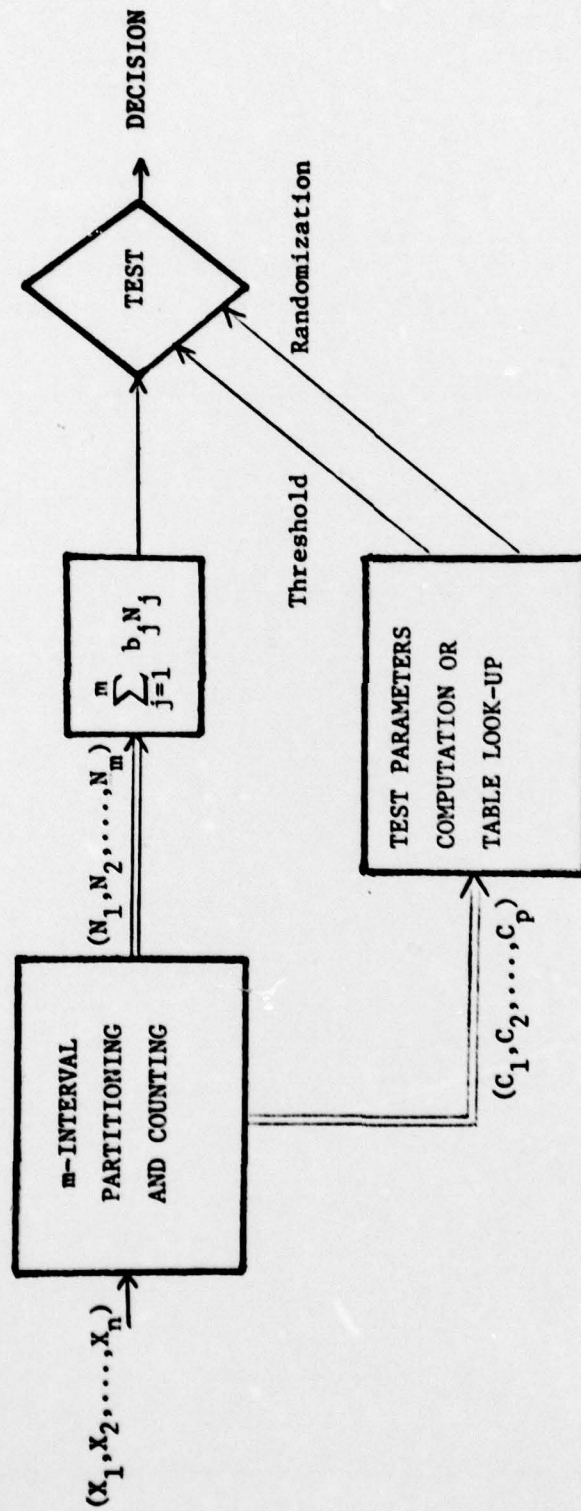


FIGURE 1

TABLE I

	$\theta/\sigma_0$	Gaussian		Double Exponential		Cauchy	
		Fixed Threshold	Conditional	Fixed Threshold	Conditional	Fixed Threshold	Conditional
N = 25 $\alpha = 0.01$ m = 4	0	0.010	0.010	0.010	0.010	0.010	0.010
	0.05	0.0178	0.0175	0.0183	0.0183	0.0176	0.0176
	0.25	0.1133	0.1077	0.1143	0.1104	0.1091	0.1046
	0.50	0.4589	0.4341	0.4296	0.3967	0.4173	0.3758
	0.75	0.8387	0.8129	0.7895	0.7333	0.7531	0.6796
	1.00	0.9805	0.9727	0.9512	0.9183	0.9208	0.8631
N = 50 $\alpha = 0.001$ m = 4	0	0.001	0.001	0.001	0.001	0.001	0.001
	0.05	0.0028	0.0027	0.0029	0.0029	0.0027	0.0027
	0.25	0.0675	0.0613	0.0665	0.0633	0.0616	0.0578
	0.50	0.5230	0.4931	0.4793	0.4366	0.4603	0.4055
	0.75	0.9458	0.9313	0.9088	0.8646	0.8765	0.8094
	1.00	0.9991	0.9985	0.9944	0.9861	0.9854	0.9616
N = 50 $\alpha = 0.001$ m = 2 (Sign Detector)	0	0.001	0.001	0.001	0.001	0.001	0.001
	0.05	0.0024	0.0024	0.0030	0.0030	0.0029	0.0029
	0.25	0.0429	0.0429	0.0652	0.0652	0.0647	0.0647
	0.50	0.3545	0.3545	0.4092	0.4092	0.4220	0.4220
	0.75	0.8266	0.8266	0.8051	0.8051	0.7965	0.7965
	1.00	0.9869	0.9869	0.9650	0.9650	0.9474	0.9474

TABLE II

Parameters of actual noise distribution				N = 25		N = 50	
$P_1$	$P_2$	$P_3$	$P_4$	$\alpha_0=0.01$	$\alpha_0=0.001$	$\alpha_0=0.01$	$\alpha_0=0.001$
0.20	0.30	0.30	0.20	0.0065	0.0005	0.0065	0.0005
0.225	0.275	0.275	0.225	0.0082	0.0007	0.0082	0.0007
0.25	0.25	0.25	0.25	0.010	0.001	0.010	0.001
0.275	0.225	0.225	0.275	0.0120	0.0014	0.0119	0.0014
0.30	0.20	0.20	0.30	0.0141	0.0018	0.0141	0.0018

TABLE III

Gaussian Noise $a_3=0.82\sigma_0$ N = 25 $\alpha_0 = 0.01$		
$\theta/\sigma_0$	Fixed Threshold	Conditional
0	0.0069	0.010
0.25	0.0909	0.1089
0.50	0.4142	0.4398
0.75	0.8132	0.8196
1.0	0.9763	0.9750

TABLE IV

$\alpha_0$	<u>a</u> = quantile vector			
	N = 25		N = 50	
	Fixed Threshold	Conditional	Fixed Threshold	Conditional
0.05	0.0438	0.0442	0.0490	0.0456
0.01	0.0092	0.0085	0.0099	0.0089
0.001	0.00078	0.00073	0.00092	0.00088